A Brief Summary of the Logical Distinctions and Relations Used in Labyrynth™

Dr. Jean W. Rioux

The Four Questions.

In one of his logical works, Aristotle notes that the kinds of thing we can know correspond to the kinds of question we can ask, of which there are four. As he says it:

> The questions we ask are equal to the things we can know, and we ask four things: whether it is so (τὸ ὅτι) why it is so (τὸ διότι) whether it is (εἰ ἔστι) and what it is (τί ἐστιν).

But these questions are not asked, nor can they be answered, in just any order whatsoever. Rather, the proper order of questioning entails asking about a thing, first, whether it is, that is, whether it exists (if that *is* a question), then what it is. Next in turn, one cannot ask (or rather, answer) the question why something is so before having already established whether it is so.

In other words, one would first need to establish whether something exists before one could reasonably inquire about the essence of that thing. (These first are what Aristotle would call simple questions, since they do entail saying one thing of another.) As for the complex questions, one could not reasonably ask for the reason for some fact before establishing the fact in the first place.

So, the proper order of questions is: whether it is (εἰ ἔστι), what it is (τί ἐστιν), whether it is so (τὸ ὅτι), why it is so (τὸ διότι).

Inference Signs.

Arguments are made up of parts (propositions), which are themselves made up of parts (subjects, predicates, and so on). For the sake of simplicity, logic texts will often express arguments according to a certain pattern (we will not concern ourselves with that exact pattern just yet). For example:

every animal is mortal
every human is an animal
∴ every human is mortal.

This argument is made up of three statements, two of which are called *premises*, while the third is the *conclusion*. In this case, we know which is which, since the symbol ∴ is taken to stand for 'therefore', which is a shorthand way of saying 'here is my conclusion'.

In cases like this it is fairly easy to distinguish an argument's conclusion from its premises. Unfortunately, arguments as they are found in everyday speaking and writing are not always so easily dealt with:

> "Could these be common porpoises?" Stanton asked. They sure had the look of the breed. But since common porpoises are found exclusively in the southern latitudes, whereas these animals had been with him for many miles as he journeyed south across the equator, they could not be common porpoises. They must belong to a similar species.

Just as in the first example, an argument is being made here. In the absence of the ∴ symbol can we identify which statements are premises and which is the conclusion? The keys to doing this are what we call *inference signs*. Inference signs are certain words we use to indicate to our listeners or readers how statements in our arguments are to be taken. The use of such signs arises from the necessity of distinguishing the different parts of arguments in order that they be understood. Consider these three statements:

human is mortal
animal is mortal
human is animal.

Without inference signs, it is impossible to decide whether this person is trying to conclude that humans are animals or that animals are mortal or that humans are mortal—in fact, there is no indication here of whether an argument is *even being made*.

Now consider this way of expressing the same three statements: since every human is an animal, every human is mortal, for every animal is mortal. This speaker is definitely making an argument: his use of the words *since* and *for* makes this clear. That is because *since* and *for* are inference signs. Here, they indicate that *every human is an animal* and *every animal is mortal* are to be taken as the premises of the argument. The proposition *every human is mortal* is seen to be the conclusion by process of elimination.

Returning to the example we gave above, we can now see that the proposition *common porpoises are found exclusively in the southern latitudes* is being taken as a premise of the argument. The same is true for the proposition which follows it, *these animals had been with him for many miles as he journeyed south across the equator*, since it is preceded by the conjunction *and* (as if one were to say since this is so and [since] that is so…). The conclusion is *they [the animals he is looking at] could not be common porpoises.*

Inference signs are useful, then, when we seek to distinguish an argument's conclusion (or, the *point* the speaker is trying to make) from its premises (the *reasons* the speaker supplies for that conclusion). Common signs of inference are *since*, *for*, and *because*, for premises and *therefore*, *thus*, *so*, and *it follows that*, for conclusions.

Argument Types.

Two of the main types of argument are categorical and conditional (or hypothetical) syllogisms. They differ in terms of the main, or foundational, premise upon which they are based.

Conditional syllogisms are based upon a conditional proposition, of the general form: *if [antecedent] then [consequent]*. By definition, the antecedent of a conditional proposition is the part following the word 'if'; the consequent is the other part, or the part following the word 'then' (if there is one—sometimes the 'then' is omitted, as in the proposition *if Cassius has a lean and hungry look, I do not trust him*).

Conditional syllogisms have three major parts: the conditional proposition (its foundational premise), a second premise (which will either affirm or deny the antecedent or the consequent found in the first premise), and a conclusion. There are four basic forms of conditional syllogism, two valid and two invalid (see Conditional Syllogisms, below, for more).

Categorical syllogisms are founded upon a categorical proposition, which is, in truth, a proposition with no 'ifs, ands, or buts' about it. Categorical propositions are of the general form: *s is p*, where *s* stands for any subject and *p* for any predicate. They, too, contain two premises: a major premise and a minor premise (see Parts of Syllogisms, below, for more), as well as a conclusion, all of them being categorical propositions.

Quality, Quantity, Type.

Categorical propositions (see Argument Types, above), that is, propositions of the general form: *s is p*, have characteristics we call *quality* and *quantity*. There are two types of quality and four types of quantity (though two of these four are of lesser interest to logicians).

All categorical propositions either affirm some predicate of a subject or deny that predicate of a subject. In their simplest forms: *s is p* is an affirmation (since it affirms *p* of *s*), while *s is not p* is a denial (since it denies that *p* belongs to *s*).

The quantity of a proposition depends upon how many members the subject is taken to refer to. Propositions which refer to an individual thing as a subject (for example, *Aristotle was Plato's student*) are said to have *singular* quantity. (Note that this proposition is also affirmative, in accordance with the distinction made above. An example of a singular proposition with negative quality is *Aristotle was not Alexander's student*.) Propositions which refer to more than a single thing as a subject can do so in a number of ways. In the proposition *every s is p*, we are referring to each and every member of the subject, whatever it may be. Such a proposition is said to have *universal* quantity. (It also is an affirmative proposition. A universal *negative* proposition has the form: *no s is p*.) Propositions of the form *some s is p* or *some s is not p* are said to have *particular* quantity (referring to a part of the whole subject). Common alternate forms of the particular negative proposition are: *not every s is p* and *every s is not p*. Finally, propositions may refer to more than a single thing as a subject, yet not determinately specify how many are being referred to. For example, while *all swans are white* is a *universal* affirmative proposition and *some swans are white* is a *particular* affirmative proposition, *swans are white* would be classified as an *indefinite* affirmative proposition (as not saying definitely either *every* or *some*).

Of the four types of propositional quantity (universal, particular, singular, and indefinite), the most commonly used (and of greatest interest to the logician) are the first two: universal and particular.

Propositional *type* is a shorthand way of referring to the four most common propositional forms: universal affirmative, universal negative, particular affirmative, and particular negative. To each is assigned a letter, thus: universal affirmative: A, universal negative: E, particular affirmative: I, and particular negative: O.

Common paradigm forms of each propositional type would be:

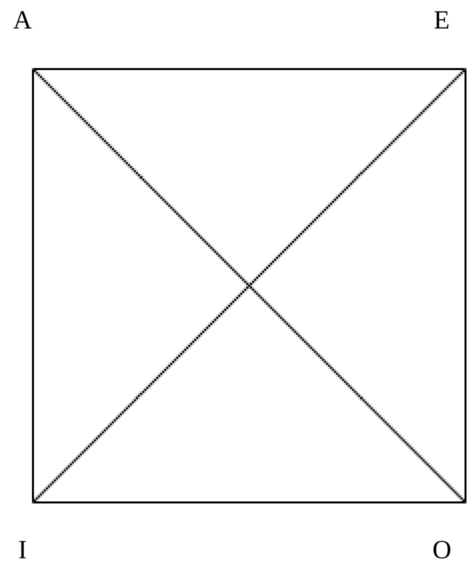
A	*every s is p, all s is p*
E	*no s is p*
I	*some s is p*
O	*some s is not p*
	not every s is p, not all s is p
	every s is not p, all s is not p

In sum, then, there are two sorts of quality, four sorts of quantity (though two of interest), and four propositional types.

Opposition.

Opposition is the name of a special relation found among certain propositions having the same subject and predicate. It occurs between affirmative and negative statements. Thus, *s is p* is the opposite of (is opposed to) *s is not p*. Taking into account the distinctions made above, logicians have devised a graphic representing the possible relations among the four propositional types. This aid is called the *square of opposition*.

By convention, universals are placed at the top corners of the square, affirmatives at the left corners, thus:



Let each of the lines on the square, then, represent the relation between any proposition and any other proposition, the topmost line representing the relation between type A and type E, for example.

According to the conditions we set down for opposites, above, then, the proposition pairs A and E, A and O, I and O, and E and I, are opposed to one another. (The relation between A and I, and E and O, respectively, is not a relation of opposition but another logical relation called *subalternation*.)

For purposes of identification, then, there are three different kinds of opposition found among the four propositional types. The opposition between universal propositions (one affirmative, one negative, so A and E) is called *contrary* opposition (and the propositions so related are said to be *contraries*). The opposition between particular propositions (one affirmative, one negative, so I and O,) is called *subcontrary* opposition (and the propositions so related are said to be *subcontraries*). Finally, there is the opposition between a universal proposition and a particular one, of which there are two instances on the square: A and O, and E and I, respectively. This is the relation called *contradictory* opposition (and these pairs of propositions are called *contradictories*).

Graphically, then, the uppermost line represents contrary opposition, the lowermost line subcontrary opposition, the diagonals contradictory opposition, and the vertical lines the relation of subalternation (which, remember, is *not* a kind of opposition).

Immediate Inferences.

Given the relations one finds on the square of opposition, it is possible to determine whether a given proposition is true, false, or undecidable (unknown) on the basis of the truth or falsity of another. Such determinations are called *immediate inferences* (as distinguished from argumentation proper, which entails an intermediate, and which are, therefore, *mediate*).

For example, if one knows that *every s is p* is true, one knows also (thereby) that *some s is not p* must be false, for the simple reason that the propositions contradict, and contradictories can be neither true nor false simultaneously. Thus, given the truth or falsity of one proposition, one can always determine the truth or falsity of its contradictory.

The same is not true for contraries, however. In this case, though the propositions cannot be true simultaneously, they can be false at the same time. Given, then, that *every s is p* is true, it immediately follows that *no s is p* must be false, and vice-versa. Given, however, that one of two contraries is false, it is impossible to say whether the other is true or false: we say it is *undecided*, then, that is, its truth or falsity is, at that point, unknown.

The rule for subcontraries is the opposite of that for contraries. Subcontraries cannot be false simultaneously, but they can be true simultaneously. Given, then, that a type I proposition is false, it would immediately follow that the corresponding type O proposition is true, and vice-versa. Given, however, that one of two subcontraries is true, the other is unknown.

One can also immediately infer on the basis of the last relation on the square, that of subalternation (between A and I and between E and O). Thus, if a given *universal* proposition is true (whether A or E), we can immediately infer that the corresponding *particular* (I or O, respectively) is also true. Similarly, if a given *particular* proposition is false (whether I or O), we can infer that the corresponding *universal* proposition (A or E, respectively) is also false. (This rule of inference is sometimes expressed as *descend with truth rise with falsity*, a clear reference to how the square of opposition itself is drawn.) All other connections are undecided.

Conditional Syllogisms.

We spoke of conditional propositions above, and of conditional syllogisms as argument forms based upon a conditional proposition as a foundation. We also mentioned that there are only four possible forms for this argument. Thus, given a conditional proposition in the foundational (first) premise, for example *if s is p then q is r*, one might then do one of four things in the second premise: affirm the antecedent, affirm the consequent, deny the antecedent, or deny the consequent. (Recall that the antecedent in a conditional proposition is that part of it following the word 'if'.) So, following the order given just now, we can see the following forms take shape:

1	*if s is p then q is r*	*(foundational first premise)*
	s is p	*(second premise affirms the antecedent)*
∴		

2	*if s is p then q is r*	*(foundational first premise)*
	q is r	*(second premise affirms the consequent)*
∴		

3	*if s is p then q is r*	*(foundational first premise)*
	s is not p	*(second premise denies the antecedent)*
∴		

4	*if s is p then q is r*	*(foundational first premise)*
	q is not r	*(second premise denies the consequent)*
∴		

Of the four, two of the forms are valid, two not. In the first form (in which we affirm the antecedent in the second premise) we can validly affirm the consequent in the conclusion, thus:

1	*if s is p then q is r*	*(foundational first premise)*
	s is p	*(second premise affirms the antecedent)*
∴	*q is r*	*(valid conclusion)*

This argument form is given the name *modus ponens* (which is often abbreviated as MP).

In the second form (in which we affirm the consequent in the second premise) we cannot validly draw any conclusion:

2	*if s is p then q is r*	*(foundational first premise)*
	q is r	*(second premise affirms the consequent)*
∴	———	*(no valid conclusion)*

To attempt to argue this way is a fallacy, specifically, the *fallacy of affirming the consequent*.

In the third form (in which we deny the antecedent in the second premise) we cannot validly draw any conclusion either:

3	*if s is p then q is r*	*(foundational first premise)*
	s is not p	*(second premise denies the antecedent)*
∴	———	*(no valid conclusion)*

To attempt to argue this way is also a fallacy, then, specifically, the *fallacy of denying the antecedent*.

Lastly, in the fourth form (in which we deny the consequent in the second premise) we can validly deny the antecedent in the conclusion, thus:

4	*if s is p then q is r*	*(foundational first premise)*
	q is not r	*(second premise affirms the antecedent)*
∴	s is not p	*(valid conclusion)*

This argument form is given the name *modus tollens* (which is often abbreviated as MT).

Distribution.

Distribution is a characteristic belonging to the terms of a proposition, that is, to its subject and its predicate. A given term is said to be distributed if it is taken to refer to the entire term, that is, to each and every member of that term; otherwise it is said to be undistributed. As to the distribution of the subjects of propositions, and particularly of types A, E, I, and O, distribution parallels quantity. Thus, the subjects of types A and E are distributed (since *all s is p* and *no s is p* are referring to each and every *s* in each case). Similarly, the subjects of types I and O are undistributed, since they are propositions with particular quantity.

As to the distribution of the predicates of propositions, it depends upon whether the proposition is affirming or denying the predicate of the subject, that is, it depends upon the proposition's quality. Affirmative propositions have an undistributed predicate, while negative ones have a distributed predicate. Thus, types A and I have undistributed predicates, while E and O have distributed predicates.

Parts of Syllogisms.

Categorical syllogisms contain three categorical propositions, as we noted above: two premises and a conclusion. Each of these parts also has parts (each of the three propositions has a subject and predicate). So, it is possible (and desirable) to be able to identify and classify categorical syllogisms on the basis of such parts.

In such an analysis, one must begin with the syllogism's conclusion. The predicate of the conclusion, by definition, is called the major term, while its subject is the minor term. Each of these terms will also be found once more in the syllogism: the major term in one of the premises, the minor term in the other premise. This gives us a way to distinguish between the premises: the one containing the major term being called the major premise, while the other premise is accordingly called the minor premise.

One part of the syllogism remains: the middle term, which, though it may appear as a subject or a predicate, is found only in the premises, not in the conclusion.

In the syllogism: all s is p, for all s is m and all m is p, then, the conclusion is stated first, followed by the minor premise (as the premise containing the minor term, s, the subject of the conclusion), then the major premise (as the premise containing the major term, p, the predicate of the conclusion).

Validity of Syllogisms.

Syllogisms which work, which successfully establish their conclusions, are said to be valid. Note that a valid syllogism is not necessary the same as one saying true things. It is possible to formulate a valid argument using true premises or from false ones; the use of false premises does not affect the argument's validity.

Thus, the syllogism: *every mammal is a reptile, every fish is a mammal, therefore every fish is a reptile* is valid (that is, it makes no logical mistakes as it moves from the premises to its conclusion), even though it contains three false statements. In its turn, the syllogism: *every station wagon is a car, because every car is a vehicle and every station wagon is a vehicle* contains three true statements, but is invalid: one cannot logically draw the conclusion from those premises without making a logical mistake.

Note that sets of rules other than this one have been drawn up for testing the validity of categorical syllogisms. Still, this rule set does successfully supply such a test.

Rule One: a syllogism can have only three terms, each used twice. (Thus, the major term is found in the conclusion and the major premise, the minor term in the conclusion and the minor premise, and the middle term in both premises and not in the conclusion.)

Rule Two: a term distributed in the conclusion must be distributed in the premise. (Note that this rule is irreversible: one need not have a term distributed in the conclusion simply because it is so in the premise.)

Rule Three: the middle term must be distributed at least once.

Rule Four: at least one of the premises must be affirmative.

Rule Five: the conclusion can be no stronger than the premises. (Affirmative is taken as being stronger than negative, universal as being stronger than particular. Thus, a syllogism with a major premise of type A, a minor premise of type I, and a conclusion of type A would be invalid for this reason. Similarly, a syllogism with a major premise of type E, a minor premise of type A, and a conclusion of type I would be invalid, since I is affirmative, and therefore stronger than E which, though universal, is also negative.)